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Estimating a Bayesian prior for steepness in Pacific rockfishes (*Sebastes* spp.) off the U.S. West Coast for the 2013 assessment cycle

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Background:
Integrated assessment models are currently the state-of-the-art tool in stock assessments. These models estimate historical biomass, exploitation rates, and productivity, propagate uncertainties from a variety of data sources, and project these levels forward given different possible management actions (Maunder and Punt 2013).

On the U.S. West Coast, most data-rich assessments use the Stock Synthesis age and length-structured modeling software (Methot and Wetzel 2013). This software estimates age-0 abundance (called 'recruits') as a function of spawning output, and penalizes annual estimates of recruitment towards a presumed stock-recruit relationship (often the Beverton-Holt; Methot and Taylor 2011). This stock-recruit relationship represents the degree of compensation in recruitment, believed to occur primarily in early juvenile survival for many Pacific rockfishes (*Sebastes* spp.; Field et al. 2010). Recruitment compensation is specifically modeled using steepness (hereafter referred to as *h*; Mace et al. 1988), a parameter ranging from 0.2 to 1 and representing the proportion of average unfished recruitment achieved on average at 20% of unfished spawning output (where *h*=0.2 represents no recruitment compensation, and *h*=1.0 represents constant recruitment for essentially all levels of spawning output).

Steepness is extremely difficult to estimate for fishes (Conn et al. 2010). Early efforts estimated a low average steepness for Pacific rockfishes (Myers et al. 1999). However, these early efforts generally used assumptions that we are now considered to be suspect, e.g., assuming that stock assessment estimates of spawning output are known without error. Subsequent efforts has also made that assumption (Dorn 2002, Forrest et al. 2010).

More recent research has relaxed this dubious assumption, specifically by developing a profile likelihood approximation to a multispecies mixed-effects model. This profile likelihood approximation is discussed in detail below. It has been used in the 2009 and 2011 assessment cycles for the U.S. West Coast to provide average values of steepness for Pacific rockfishes. In this document, we explain efforts to replicate previous 2011 results, improvements to the software and database, and results for the 2013 assessment cycle.

Methods and Results

Profile likelihood methods

Mixed-effects models are useful when estimating parameters from one or more species as arising from a similar process with a common distribution of likely values (Gelman and Hill 2007, Zuur et al. 2009). Maximum marginal likelihood mixed-effects models are often estimated using empirical Bayes methods:

$$L(\theta,\phi \mid Data) = \int_{\varepsilon} L(\theta,\varepsilon \mid Data)\cdot p(\varepsilon \mid \phi)\cdot \mathrm{d}\varepsilon \qquad \text{(Eq. 1)}$$

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where θ is a vector of fixed-effects, ϕ is a vector of hyperparameters, *Data* is the data set, ε is a vector of random-effect coefficient, $p(\varepsilon \mid \phi)$ is the hyperdistribution for ε, and the integral across ε is the empirical Bayes treatment of random effects.

We are interested in estimating an average value for *h*, which we seek to treat as a random effect. Ongoing research has demonstrated that maximum marginal likelihood models (Eq. 1) can be approximated by using a marginal "profile" likelihood (Punt et al. in preparation, Thorson et al. In review):

$$\int_{\varepsilon} L(\theta, \varepsilon \mid Data) \cdot p(\varepsilon \mid \phi) \cdot \mathrm{d}\varepsilon \approx \int_{\varepsilon} C(\varepsilon \mid Data) \cdot p(\varepsilon \mid \phi) \cdot \mathrm{d}\varepsilon \qquad \text{(Eq. 2)}$$

where $C(\varepsilon \mid Data)$ is the profile likelihood equal to the value of $L(\theta,\varepsilon \mid Data)$ when θ is set to its maximum likelihood value for a given ε. This maximum marginal profile likelihood (Eq. 2) is easily obtained using existing stock assessment software and assessment files, and provides a computationally tractable approximation to the marginal likelihood model.

Steepness database in 2011 and 2013

The 2013 database has all 16 stocks that were previously included in the 2011 database, plus an additional 4 more stocks (Table 1). Seven of the 16 previous stocks have been updated using more recent assessment versions. All stocks were re-run using the NWFSC archive and results were checked against the 2011 database. In two cases, the 2011 database had minor errors (Greenstriped and Black, northern stock), and in two cases the NWFSC archive had incorrect files (northern stock of Black, and chilipepper).

'Raw' likelihood profiles for the 2013 database are shown in Fig. 1, and generally agree with profiles previously seen in the 2011 database.

Changes to the 2011 software

The 2013 software contains the following changes the previous 2011 software:

1. I now use a uniform prior on the average steepness for rockfish as a group. The 2011 software by contrast had an implicit prior:

   $$\Pr[h] \sim \frac{1}{h^*(1-h^*)}$$

   where

   $$h^* = (h - 0.2)/0.8$$

2. I use a logit-normal hyperdistribution for steepness. The 2011 software by contrast had a normal distribution in logit-space (i.e., did not include the Jacobian to account for using logit-space).

The 2013 software continues to use the same informative prior on the standard deviation of between-stock variability in logit-space, as was used in the 2011 software.

Replicating 2011 results and generating 2013 results

I first replicated results from the previous 2011 cycle, using Martin's 2011 software and 2011 database (Fig. 2), and confirmed that these are identical to results used in the 2011 assessment cycle. I next generated results using Martin's 2011 database but using the 2013 software (Fig. 3), and confirmed that these results are identical to the 2011 results (to approximately 0.001).

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Next, I generated results using the 2013 software and all 20 species in the 2013 database (Fig. 4). This resulted in a small increase in the mean of the prior distribution (0.78 instead of 0.75), and an increase in the dispersion of the prior distribution (0.197 instead of 0.166).

I then solicited input from the SSC. After discussion, the SSC decided to restrict analysis to the 10 tier-1 species (black, bocaccio, canary, chilipepper, darkblotched, gopher, Pacific Ocean perch, splitnose, widow, and yellowtail) as listed by O. Hamel (pers. comm.). Both northern and southern assessments of black rockfish were used, although the log-likelihood for each was given a 0.5 weighting in the objective function, to ensure that they together had an equal weighting to the other species. 'Raw' likelihood profiles for these 11 stocks in the 2013 database are shown in Fig. 5. Application of the 2013 software to these 11 stocks had no effect on the mean of the prior distribution (still 0.78) and caused a decrease in the dispersion of the prior distribution (0.152 instead of 0.197).

Acknowledgements

Thank you to A. Punt and M. Dorn for sharing their previous software, M. Dorn for sharing his previous database, S. Miller for helping to build the 2013 database, the many assessment authors for their previous work and ongoing help in resolving questions, and the SSC for helpful discussions. Particular thanks go to A. Punt, M. Dorn, J. Hastie, and O. Hamel for help in planning this project.

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Table 1 – List of stocks, assessment year, and assessment author used in the previous 2011 database and the updated 2013 database.

	2011 database		2013 database	
Stock	**Year**	**Author**	**Year**	**Author**
Greenstriped	2009	Allan Hicks, Melissa Haltuch, Chantel Wetzel	2009	Allan Hicks, Melissa Haltuch, Chantel Wetzel
Bocaccio	2009	John Field, EJ Dick, Don Pearson, Alec MacCall	2011	John Field
Canary	2009	Ian Stewart	2011	John Wallace, Jason Cope
Darkblotched	2009	John Wallace, Owen Hamel	2011	Andi Stephens, Owen Hamel, Ian Taylor, Chantel Wetzel
POP	2009	Owen Hamel	2011	Owen Hamel, Kotaro Ono
Widow	2009	Xi He, Don Pearson, EJ Dick, John Field, Stephen Ralston, Alec MacCall	2011	Xi He, Don Pearson, EJ Dick, John Field, Stephen Ralston, Alec MacCall
Yelloweye	2009	Ian Stewart, John Wallace, Carey McGilliard	2011	Ian Taylor, Chantell Wetzel
Splitnose	2009	Vlada Gertseva, Jason Cope, Donald Pearson	2009	Vlada Gertseva, Jason Cope, Donald Pearson
Black_N	2007	Farron R. Wallace, Yuk Wing Cheng, and Tien-Shui Tsou	2007	Farron R. Wallace, Yuk Wing Cheng, and Tien-Shui Tsou
Black_S	2007	David B. Sampson	2007	David B. Sampson
Blue	2007	Meisha Key, Alec MacCall, John Field, Debbie Aseltine-Neilson, Kirk Lynn	2007	Meisha Key, Alec MacCall, John Field, Debbie Aseltine-Neilson, Kirk Lynn
Chillipepper	2007	John Field	2007	John Field
Cowcod	2007	E.J. Dick, Stephen Ralston, and Don Pearson	2009	EJ Dick, Stephen Ralston, Don Pearson, John Wiedenmann
Yellowtail	2000	Jack Tagart, Farron Wallace, Jim Ianelli	2000	Jack Tagart, Farron Wallace, Jim Ianelli
Vermilion_N	2005	Alec D. MacCall	2005	Alec D. MacCall
Gopher	2005	Meisha Key, Alec MacCall, Traci Bishop, Bob Leos	2005	Meisha Key, Alec MacCall, Traci Bishop, Bob Leos

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Greenspotted_N	NA	[Not previously included]	2011	E.J. Dick, Don Pearson, and Steve Ralston
Greenspotted_S	NA	[Not previously included]	2011	E.J. Dick, Don Pearson, and Steve Ralston
Blackgill	NA	[Not previously included]	2011	John C. Field and Don Pearson
Vermillion_S	NA	[Not previously included]	2005	Alec D. MacCall

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Fig. 1 – Likelihood profiles for all species listed in Table 1

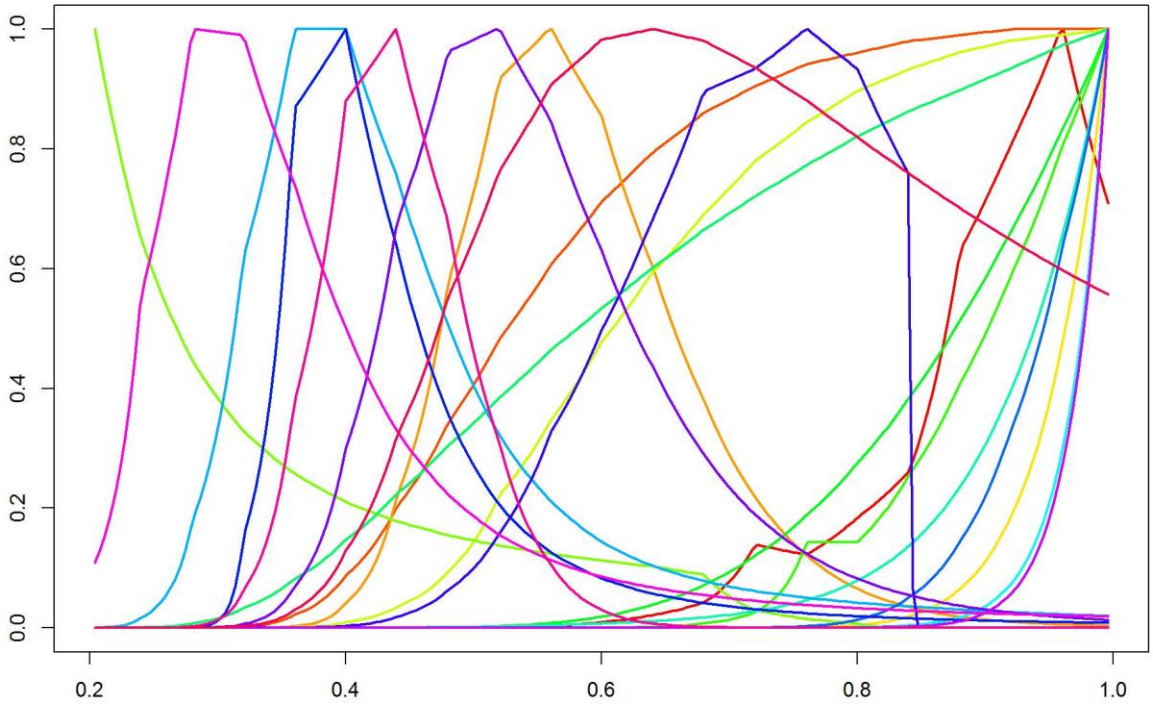

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Fig. 2 – Replication of Martin's (2011) profile using Martin's (2011) database and 2011 code, showing (A) posterior distribution for "mu_h", i.e. the mean steepness in nominal-space, (B) "sd_beta", i.e. the standard deviation of steepness in logit-space, (C) their bivariate posterior, and (D) the predictive distribution for steepness.

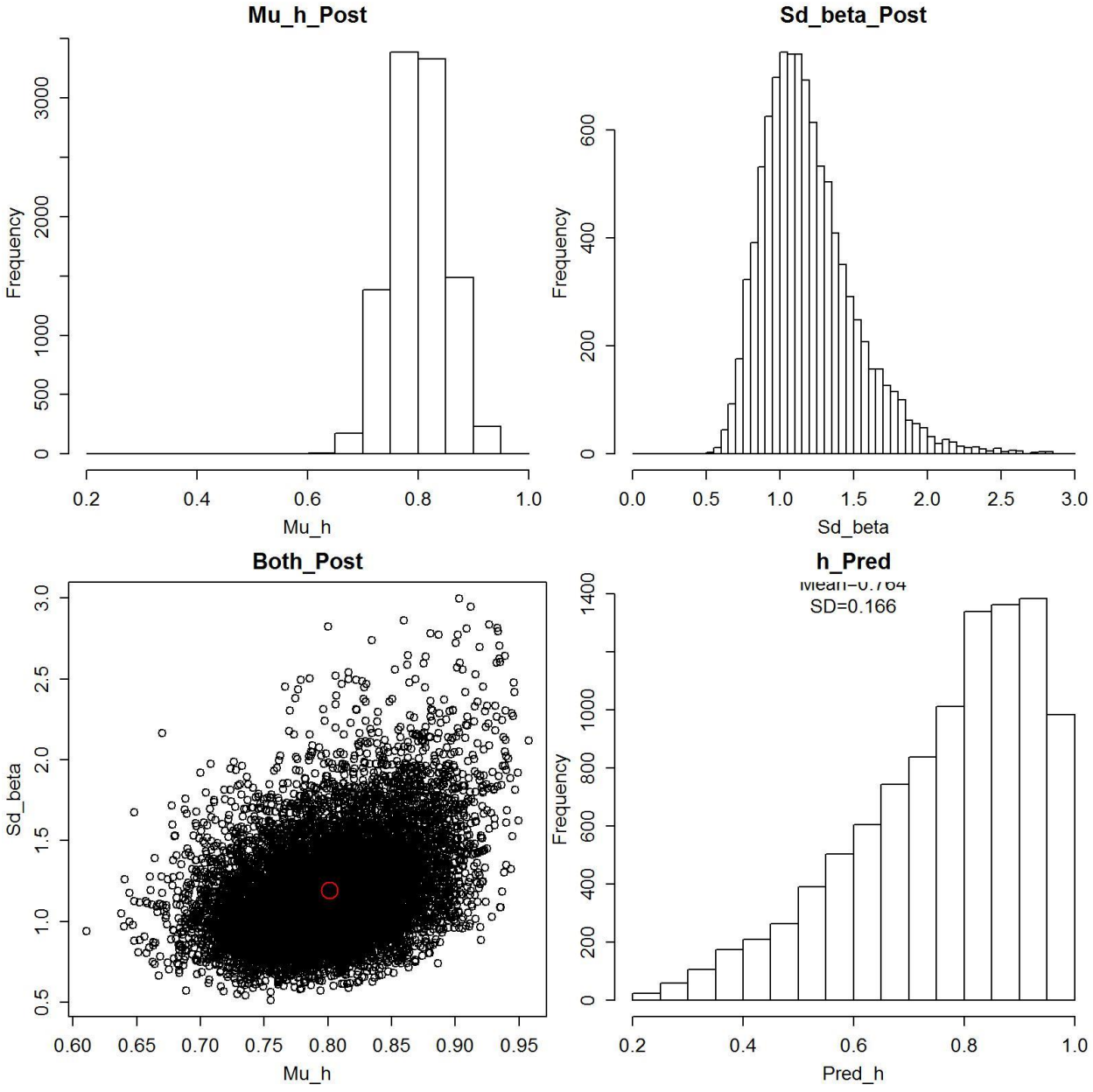

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Fig. 3 – Replication of Martin's (2011) profile using Martin's (2011) database and updated (2013) code, showing (A) posterior distribution for "mu_h", i.e. the mean steepness in nominal-space, (B) "sd_beta", i.e. the standard deviation of steepness in logit-space, (C) their bivariate posterior, and (D) the predictive distribution for steepness.

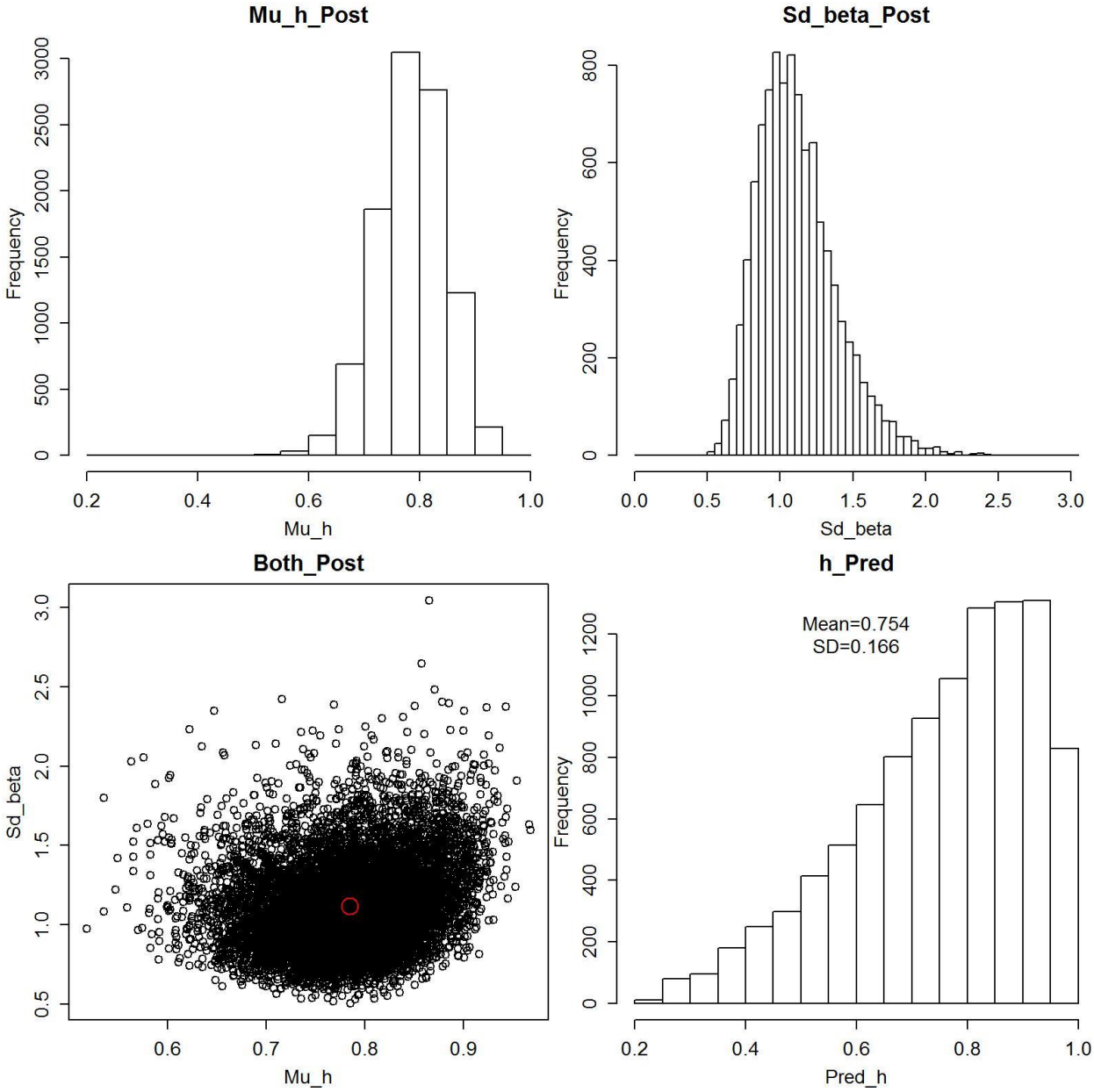

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Fig. 4 – Preliminary prior using updated (2013) database and updated (2013) code, showing (A) posterior distribution for "mu_h", i.e. the mean steepness in nominal-space, (B) "sd_beta", i.e. the standard deviation of steepness in logit-space, (C) their bivariate posterior, and (D) the predictive distribution for steepness.

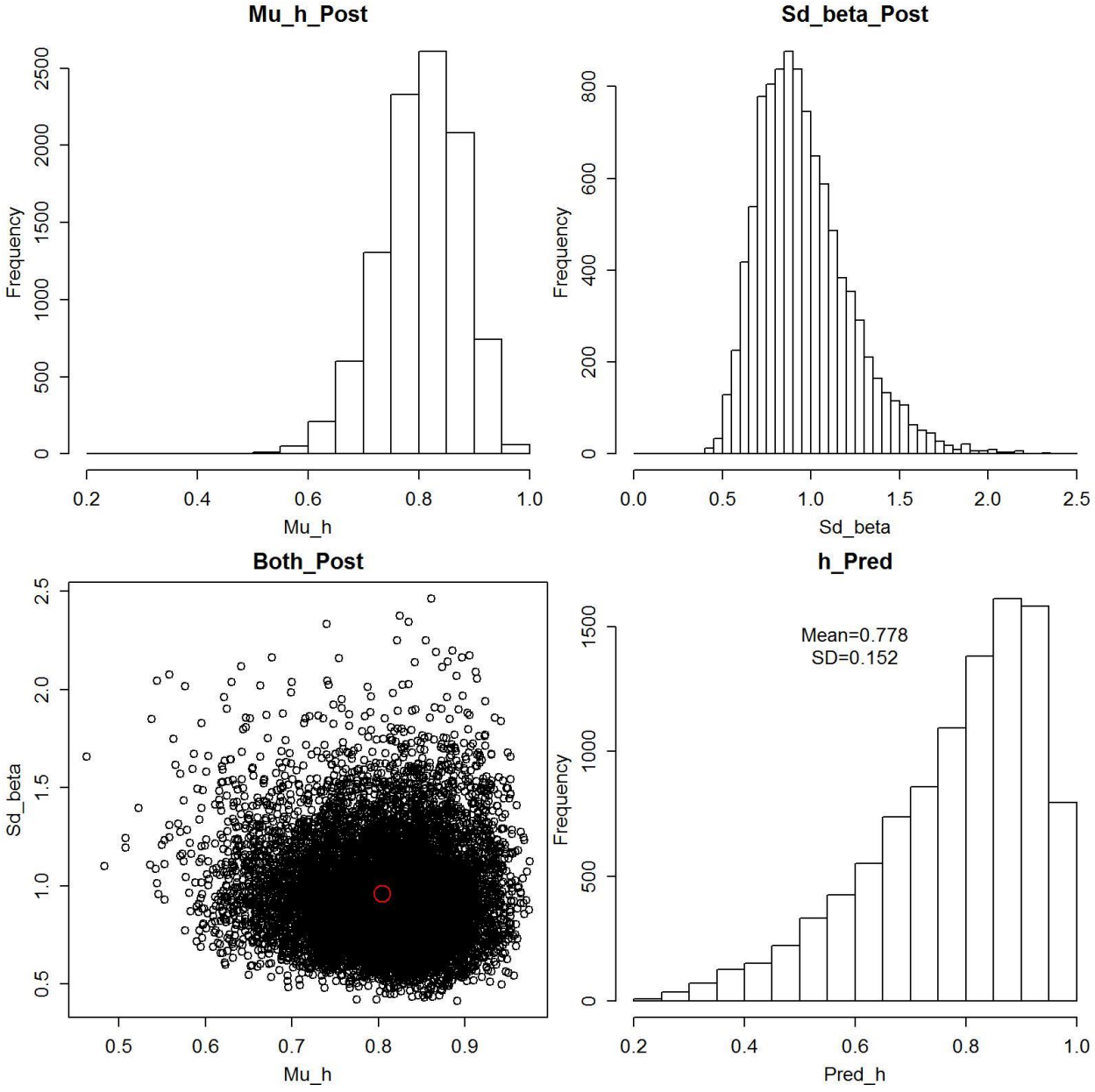

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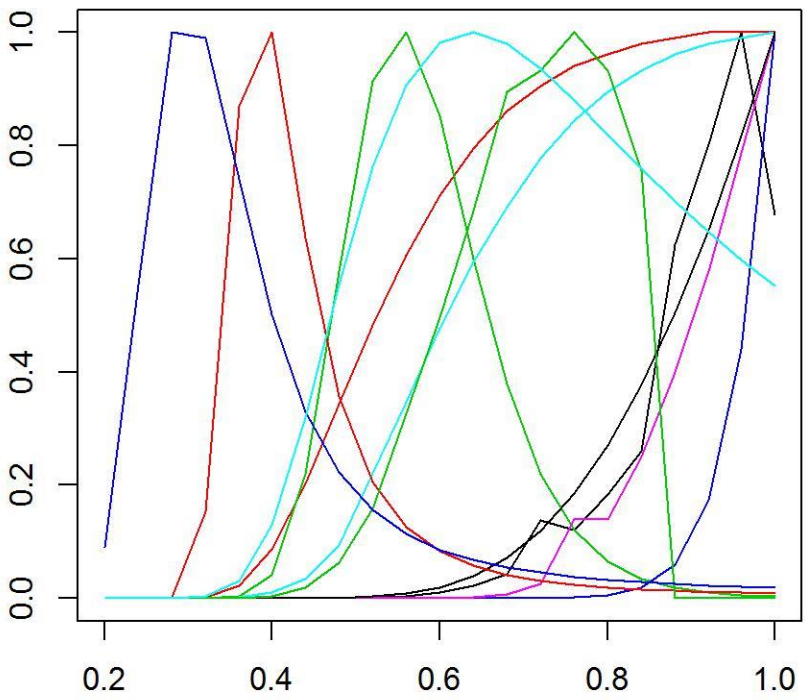
Fig. 5 – Likelihood profiles for all Tier 1 assessments listed in Table 1 and ultimately used to estimate the 2013 steepness prior.



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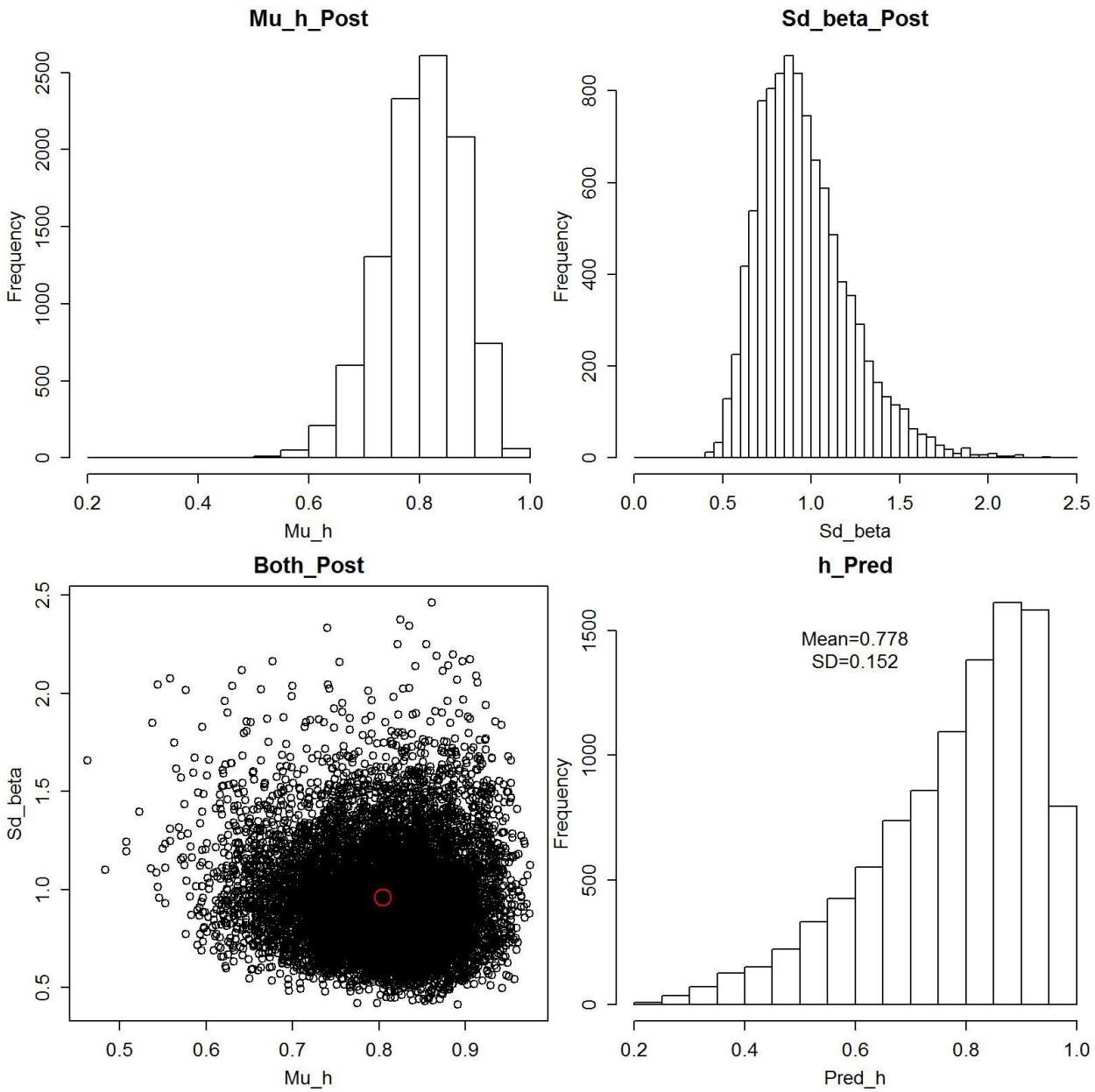
Fig. 6 – Finalized prior using 11 Tier 1 stocks from the updated (2013) database and updated (2013) code, showing (A) posterior distribution for "mu_h", i.e. the mean steepness in nominal-space, (B) "sd_beta", i.e. the standard deviation of steepness in logit-space, (C) their bivariate posterior, and (D) the predictive distribution for steepness.